Finite temperature crossovers near quantum tricritical points in metals

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We present a renormalization-group study of quantum tricriticality in metals. We evaluate the correlation length in the quantum critical region of the phase diagram, extending into finite temperatures above the quantum critical or tricritical point. We calculate the finite temperature phase boundaries and analyze the crossover behavior when the system is tuned between quantum criticality and quantum tricriticality. Interactions between order-parameter fluctuations are usually overshadowed by generic (noncritical) temperature dependences in the quantum tricritical regime.

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I. INTRODUCTION

Quantum critical behavior in metals is a topic of prime interest for theorists and experimentalists in condensed-matter physics.[1–7] Despite significant effort several intriguing puzzles remain unresolved by the quickly developing theory of quantum phase transitions. These include both, fundamental problems regarding the correct low-energy action to describe quantum critical systems and the physical mechanisms responsible for unusual behavior of specific compounds.

In the conventional scenario of quantum criticality the critical temperature T_c for a second-order transition is driven to zero by a tuning parameter, so that a quantum critical point emerges. However, it is also possible that below a tricritical temperature the transition becomes first order. Then a particular brand of quantum criticality arises when the finite temperature (T) phase boundary terminates at $T=0$ with the tricritical point. Strictly speaking, in real systems this scenario is hardly feasible since it requires fine tuning of two nonthermal parameters. However, proximity to quantum tricriticality has been invoked to explain a number of experimentally observed properties of metallic compounds.[8–13]

Although the relevance of quantum tricriticality was emphasized in a number of recent works,[10–13] a renormalization-group (RG) study addressing crossovers between critical and tricritical behaviors, in particular, in the quantum critical, non-Fermi-liquid regime, is missing. In this paper we present an analysis of a model system displaying such crossovers. We build upon an earlier work,[14] where renormalization-group flows of a quantum ϕ^6 model were already studied. The main focus of that work was to show that order-parameter fluctuations may turn first-order transitions occurring within the bare model into continuous transitions. Here we derive an analytical solution to linearized RG equations, which allows us to describe critical and tricritical behavior and to specify crossover temperatures.

II. MODEL AND RG SETUP

Let us consider the different scenarios which are envisaged in the following. In Fig. 1(a) we schematically depict a generic phase diagram of a system exhibiting a Gaussian quantum critical point (QCP). At $T=0$ the system can be tuned between an ordered and a disordered state by varying a nonthermal control parameter r. At finite T, in the disordered phase, a crossover region, schematically represented here with a line, separates the Fermi-liquid and quantum critical regimes. For $r<0$, the T_c line separates the phases of the system at finite T. The classical critical region, where non-Gaussian fluctuations occur, is bounded by the Ginzburg lines and vanishes as $T\to 0$. By varying another system parameter the phase diagram can be continuously deformed so that the transition becomes first order at sufficiently low T, below a tricritical temperature T^{tri} [see Fig. 1(c)]. For a particular value of this parameter the tricritical point is located exactly at $T=0$ so that the transition is second order for all $T>0$ but the scaling properties of the system for $T\to 0$ are governed by the quantum tricritical point (QTCP) rather than the quantum critical point. This special scenario displaying a different scaling behavior is depicted in Fig. 1(b). The purpose of the paper is to compute how the crossovers between the scenarios of Fig. 1(a) through Fig. 1(b) to Fig. 1(c) occur. Even in scenario (a), a "hidden" tricritical point, i.e., the vicinity to a first-order transition, can affect the scaling behavior at higher temperatures.

The conventional description of quantum criticality, which we shall rely upon here, invokes the Hertz action.[15] This describes a bosonic mode overdamped by particle-hole excitations across the Fermi level and is applicable under the assumption that the electronic degrees of freedom may be integrated out. The original framework of Hertz[15] and Millis[16] was recently extended to account for a number of systems and phenomena not covered by the original studies. These include field-tuned quantum critical points,[17] metamagnetic transitions,[18,19] phase transitions induced by a nonequilibrium drive[20] as well as dimensional crossovers,[21] and quantum criticality involving multiple time scales.[22] We focus on the case of discrete symmetry breaking, which can be described by a scalar order-parameter field ϕ and an action of the form

$$S[\phi] = \frac{1}{2}\int_p \phi_p\left(Z_\omega\frac{|\omega_n|}{|\mathbf{p}|^{z-2}} + Z\mathbf{p}^2\right)\phi_{-p} + \mathcal{U}[\phi]. \tag{1}$$

Here ϕ_p with $p=(\mathbf{p},\omega_n)$ is the momentum representation of the order-parameter field, where $\omega_n=2\pi nT$ with integer n

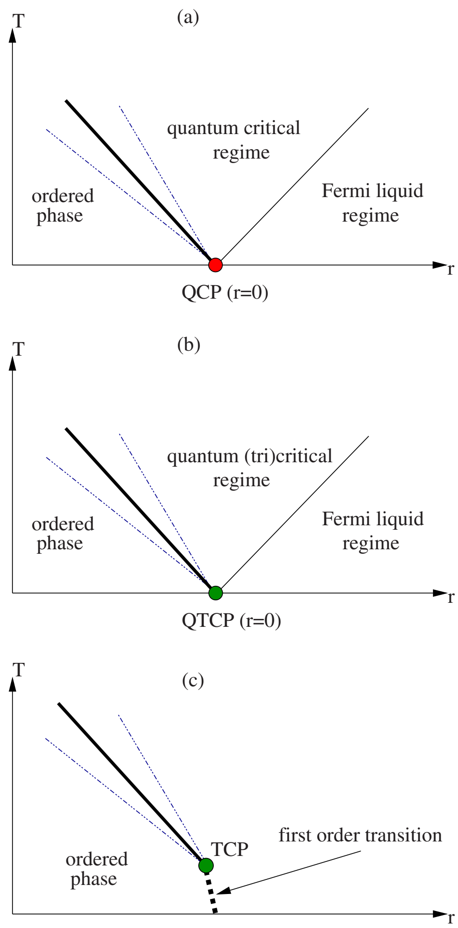

FIG. 1. (Color online) Generic phase diagrams for temperature T and tuning parameter r, where a system exhibits (a) a QCP, (b) a QTCP, and (c) a TCP at finite temperature. Varying another control parameter one can tune a system between the depicted situations (a)–(c). Bold solid (dotted) lines mark second- (first-) order phase transitions, thin solid lines indicate the crossover between quantum critical and Fermi-liquid behavior. The thin dotted lines indicate the boundaries of the Ginzburg region.

denotes the (bosonic) Matsubara frequencies. We have defined $\int_p = T\Sigma_{\omega_n}\int \frac{d^dp}{(2\pi)^d}$. The action is regularized in the ultraviolet by restricting momenta to $|\mathbf{p}| \leq \Lambda_0$. The term $\frac{|\omega_n|}{|\mathbf{p}|^{z-2}}$, where $z \geq 2$ is the dynamical exponent, effectively accounts for overdamping of the order-parameter fluctuations by fermionic excitations across the Fermi surface.

$\mathcal{U}[\phi]$ is a local effective potential,

$$\mathcal{U}[\phi] = \int_0^{1/T} d\tau \int d^dx U[\phi(x,\tau)], \tag{2}$$

which we expand to sixth order in ϕ

$$U(\phi) = a_2\phi^2 + a_4\phi^4 + a_6\phi^6. \tag{3}$$

This ansatz assumes the absence of a field explicitly breaking the inversion symmetry. We require $a_6 > 0$ to stabilize the system at large $|\phi|$. The bare effective potential yields the well-known phase diagram,[23] exhibiting a second-order transition for $a_4 > 0$, a first-order transition for $a_4 < 0$, and a tricritical point at $a_2 = a_4 = 0$. The mass parameter a_2 plays the role of the nonthermal control parameter (r) to tune the transition. By varying the quartic coupling a_4 one can deform the phase diagram so that the transition is second or first order at low T (compare Fig. 1).

In conventional quantum criticality, the temperature dependence of the coefficients a_2, a_4, and a_6 can be neglected, as it leads only to subleading corrections. However, it turns out that in a quantum tricritical regime the generic quadratic temperature dependence of these coefficients yields a dominant contribution to the temperature dependence of the correlation length.

The flow equations are obtained employing the one-particle irreducible version of the functional RG.[24–27] The present RG truncation is perturbative and all of the results of this work can equally well be obtained using other RG schemes. The derivation of the flow equations is described in Ref. 14, where the present quantum ϕ^6 model was applied to analyze the effect of fluctuations on the order of quantum phase transitions and the shapes of the finite T phase boundaries. The approximation applied amounts to assuming that the effective potential preserves the form given by Eq. (3) with flowing couplings and that the propagator retains its initial form. The present study deals with situations where the quantum critical point is Gaussian. We disregard the flow of Z, which is equivalent to neglecting the anomalous dimension of the order-parameter field. Analogous truncations retaining the flow of Z were applied in Refs. 14 and 28. The results of Ref. 28 indicate that universal aspects of the shape of the phase boundary are not affected by non-Gaussian thermal fluctuations. We also neglect the renormalization of the factor Z_ω. The flow of Z_ω was computed in Ref. 28 and shown to be negligible. The present truncation reproduces the essential features of the system except for the narrow vicinity of the second-order transition at $T > 0$, where nonetheless the shape of the phase boundary is described correctly. Note, however, that the approach can be adapted to capture the anomalous dimension of the order-parameter field[28] and to deal with cases where the fixed point associated with the quantum critical point is not Gaussian.[29]

Relying on the approximations described above, the flow is described by a set of three ordinary differential equations for the couplings a_2, a_4, and a_6 as functions of the cut-off scale Λ. In practice, whenever the effective potential features nonzero minima at $\pm\phi_0$, we find it more convenient to write the flow equations in terms of the variables $\rho_0 = \frac{1}{2}\phi_0^2$, a_4, and a_6. The coupling a_2 is then obtained from

$$a_2 = -4\rho_0(a_4 + 3a_6\rho_0). \tag{4}$$

As long as there exists a nonzero ρ_0, the evolution of the effective potential is given by the flow equations derived in Ref. 14,

$$\partial_t\rho_0 = 2v_d Z^{-1}\Lambda^{d-2}\left(3 + 2\frac{6a_6\rho_0}{6a_6\rho_0 + a_4}\right)l_1^d, \tag{5}$$

$$\partial_t a_4 = 12 v_d \Lambda^d \left[\frac{4}{3} l_2^d \frac{(30 a_6 \rho_0 + 3 a_4)^2}{Z^2 \Lambda^4} - 5 l_1^d \frac{a_6}{Z\Lambda^2} \right] - 6 \rho_0 \partial_t a_6, \tag{6}$$

$$\partial_t a_6 = 16 v_d \Lambda^d \left[-\frac{8}{3} l_3^d \frac{(30 a_6 \rho_0 + 3 a_4)^3}{Z^3 \Lambda^6} + 15 l_2^d a_6 \frac{30 a_6 \rho_0 + 3 a_4}{Z^2 \Lambda^4} \right], \tag{7}$$

where $v_d^{-1} = 2^{d+1} \pi^{d/2} \Gamma(d/2)$ and $t = \log(\Lambda/\Lambda_0) \le 0$. The threshold[24] functions $l_i^d(\delta)$ are determined from

$$l_0^d(\delta) = \frac{1}{4} v_d^{-1} \Lambda^{-d} \int_p \frac{\partial_t R^\Lambda(\mathbf{p})}{Z_\omega \frac{|\omega_n|}{|\mathbf{p}|} + Z\mathbf{p}^2 + R^\Lambda(\mathbf{p}) + Z\Lambda^2 \delta}, \tag{8}$$

$$l_1^d(\delta) = -\frac{\partial}{\partial \delta} l_0^d(\delta), \tag{9}$$

$$l_n^d(\delta) = -\frac{1}{n-1} \frac{\partial}{\partial \delta} l_{n-1}^d(\delta), \quad n \ge 2. \tag{10}$$

In Eqs. (5)–(7), $\delta = \delta(\rho) = \frac{1}{Z\Lambda^2}[U'(\rho) + 2\rho U''(\rho)]$ and the threshold functions are evaluated at the value of δ corresponding to ρ_0, that is, $l_n^d = l_n^d(\delta)_{\rho=\rho_0}$.

For $\rho_0 = 0$ the flow equations are simplified as the interaction vertices involving an odd number of legs vanish. In terms of the couplings a_2, a_4, and a_6 they read

$$\partial_t a_2 = -24 v_d \frac{a_4}{Z\Lambda^{2-d}} l_1^d, \tag{11}$$

$$\partial_t a_4 = v_d \Lambda^d \left[\left(12 \frac{a_4}{Z\Lambda^2} \right)^2 l_2^d - 60 \frac{a_6}{Z\Lambda^2} l_1^d \right], \tag{12}$$

$$\partial_t a_6 = v_d \Lambda^d \left[-\frac{2}{3} \left(12 \frac{a_4}{Z\Lambda^2} \right)^3 l_3^d + 12 \frac{a_4}{Z\Lambda^2} \left(60 \frac{a_6}{Z\Lambda^2} \right) l_2^d \right]. \tag{13}$$

III. RESULTS

Essential features of the solutions to the equations provided in the previous section (the values of the exponents describing the system in the quantum critical regime, in particular) can be computed analytically by taking only the dominant terms in the flow equations into account. Here we analyze the linearized version of the flow equations in spatial dimension $d > 2$. We shall use the following rescaled variables:

$$\delta = \frac{2 a_2}{Z\Lambda^2}, \quad u = \frac{a_4}{Z^2 \Lambda^{4-d}}, \quad v = \frac{a_6}{Z^3 \Lambda^{6-2d}}, \quad \tilde{T} = \frac{2\pi T Z_\omega}{Z\Lambda^z}. \tag{14}$$

We linearize the flow equations around the zero-temperature Gaussian fixed point

$$\partial_t \delta = -2\delta - 48 v_d l_1^d(\delta) u, \tag{15}$$

$$\partial_t u = (d-4) u - 60 v_d l_1^d(\delta) v, \tag{16}$$

$$\partial_t v = (2d - 6) v. \tag{17}$$

After this step, the terms involved are analogous to those analyzed by Millis,[16] where $v = 0$, so that only mass renormalization from the tadpole diagram is considered.

In the above form we still find the flow equations hard to solve by hand as the threshold function $l_1^d(\delta)$ involves integrals [see Eqs. (8) and (9)]. Inspired by the seminal work by Millis,[16] we additionally expand the function l_1^d for $\tilde{T} \ll 1$ and $\tilde{T} \gg 1$. In the regime $\tilde{T} \ll 1$ the flow is dominated by quantum physics while for $\tilde{T} \gg 1$ the quantum contributions to the flow equations are negligible compared to the classical ones. Similarly to Ref. 16 we shall integrate the flow equations from $\Lambda = \Lambda_0$ to $\Lambda = \Lambda^*$ determined by $\tilde{T}(\Lambda^*) = 1$ using the equations valid strictly speaking for $\tilde{T} \ll 1$. The result will then serve as the initial condition for the solution of the flow from Λ^* to $\Lambda = 0$, where we shall, in turn, use the asymptotic flow equations valid in the classical sector $\tilde{T} \gg 1$. As we checked by a direct numerical solution to Eqs. (5)–(7) and (11)–(13), this approximation has little impact on the obtained values of the nonuniversal quantities. The critical exponents are not affected by it at all.

Performing the above-mentioned expansion of the threshold function l_1^d we find for $\tilde{T} \ll 1$

$$\partial_t \delta \approx -2\delta - 48 v_d \frac{4T}{\tilde{T}} \frac{u}{d+z-2}, \tag{18}$$

$$\partial_t u \approx (d-4) u - 60 v_d \frac{4T}{\tilde{T}} \frac{v}{d+z-2} \tag{19}$$

while for $\tilde{T} \gg 1$

$$\partial_t \delta \approx -2\delta - 48 v_d \frac{2T}{d} u, \tag{20}$$

$$\partial_t u \approx (d-4) u - 60 v_d \frac{2T}{d} v. \tag{21}$$

The above equations, together with Eq. (17), form a set of linear ordinary differential equations of first order. Equation (17) is solved immediately. Plugging the result into the flow of u and integrating, we calculate $u(\Lambda)$ for $\Lambda > \Lambda^*$ and $\Lambda < \Lambda^*$. Demanding that $a_4^r \equiv \lim_{\Lambda \to 0} a_4(\Lambda) = \lim_{\Lambda \to 0} Z^2 \Lambda^{4-d} u(\Lambda) > 0$, we find a condition for the occurrence of a second-order transition. If $a_4^r(T) = 0$ at some $T \ge 0$, there is a tricritical point at this T. We find that the transition is always second order at sufficiently high T. It may turn first order as T approaches zero. The condition $a_4^r > 0$ yields

$$u_0 > -v_0 (C + C' T^{(d+z-2)/z}), \tag{22}$$

where $u_0 = u(\Lambda = \Lambda_0)$, $v_0 = v(\Lambda_0)$, and C, C' are positive constants (depending on Λ_0, Z_ω, Z, d, and z). The explicit expressions for these constants are given in the appendix. If the condition for u_0 given by Eq. (22) is fulfilled for $T=0$, it clearly holds for any $T\geq 0$. In such case there is a line of critical points terminating at $T=0$ with a QCP. This is the scenario (a) in Fig. 1. Note that Eq. (22) can hold for negative values of u_0, but is never violated for $u_0>0$. This provides an explanation for the observation of Ref. 14 that fluctuations may turn first-order quantum phase transitions continuous but not vice versa in this model. For $v_0=0$ one obviously recovers $u_0>0$ and the standard quantum critical behavior. A quantum tricritical point exists if $u_0=u_0^{\mathrm{tri}}=-Cv_0$, which corresponds to the scenario (b) in Fig. 1. In case the condition Eq. (22) is violated at $T=0$, it is still always fulfilled at sufficiently high temperatures, namely, for $T>T^{\mathrm{tri}}$, where

$$T^{\mathrm{tri}}=[-(u_0/v_0+C)/C']^{z/(d+z-2)}. \tag{23}$$

In this case the transition becomes first order for $T<T^{\mathrm{tri}}$, which is scenario (c) in Fig. 1.

We now discuss the flow of δ. Plugging the solution obtained for $u(\Lambda)$ into Eqs. (18) and (20), we integrate Eq. (18) from Λ_0 to Λ^*. The solution at Λ^* is the initial condition for Eq. (20). In cases where critical behavior occurs, the correlation length can be extracted as $\xi^{-2}\propto \lim_{\Lambda\to 0}\Lambda^2\delta$. The result has the following structure:

$$\xi^{-2}=D_0(T)+D_1T^{(d+z-2)/z}+v_0D_2T^{[2(d+z)-4]/z}, \tag{24}$$

where D_0 depends on δ_0, u_0, and v_0 while D_1 and D_2 do not depend on δ_0. Notice that $D_1=\bar{D}_1(u_0-u_0^{\mathrm{tri}})$, where $\bar{D}_1>0$. Explicit expressions are presented in the appendix. The generically quadratic temperature dependence of the bare model parameters δ_0, u_0, and v_0 for low T induces a corresponding quadratic temperature dependence of D_0, D_1, and D_2. While the temperature dependence of D_1 and D_2 is always negligible, the quadratic T dependence of $D_0(T)=D_0+D_0'T^2$ dominates in the tricritical regime (see below) and must therefore be kept.[12,13]

Criticality occurs at $T=0$ when $\delta_0=\delta_0^{\mathrm{cr}}$ such that $D_0=0$. At finite T the behavior of ξ^{-2} is governed by several terms. The quantum critical term $D_1T^{(d+z-2)/z}$ dominates for sufficiently low T, provided $D_1>0$, and reproduces the result for $\xi^{-2}(T)$ derived already by Millis.[16] Compared to that term, the quadratic temperature dependence from $D_0(T)$ and the term $v_0D_2T^{[2(d+z)-4]/z}$ coming from the ϕ^6 interaction are subleading corrections. On the other hand, if $u_0=u_0^{\mathrm{tri}}$, the coefficient D_1 becomes zero and the correlation length exhibits a different behavior,

$$\xi^{-2}=D_0'T^2+v_0D_2T^{[2(d+z)-4]/z}. \tag{25}$$

The exponent of the second term is 3 for $z=2$, $d=3$ and 8/3 for $z=3$, $d=3$. The latter exponent was obtained already from a self-consistent fluctuation resummation.[11] At low temperatures the "trivial" quadratic term thus dominates the temperature dependence of ξ in the tricritical regime. The relevance of the quadratic temperature dependences of the bare parameters in the tricritical regime was noticed already by Misawa *et al.*[12,13] However, the exponent for the temperature dependence of the order-parameter susceptibility obtained by these authors for the quantum tricritical regime turned out to be identical to that for conventional quantum criticality, that is, 3/2 for $z=2$. That result, obtained from a self-consistent summation of staggered and homogeneous fluctuations, is clearly at variance with our result.[30]

For quantum critical systems close to quantum tricriticality (small D_1), the correlation length follows a tricritical temperature dependence above a crossover temperature

$$T_{\mathrm{cross}}=\left(\frac{D_1}{D_0'}\right)^{z/(2+z-d)}. \tag{26}$$

For the special case of a very small D_0', tricritical behavior with an exponent $\frac{2(d+z)-4}{z}$ is observed for temperatures above

$$T_{\mathrm{cross}}=\left(\frac{D_1}{v_0D_2}\right)^{z/(d+z-2)}. \tag{27}$$

Equation (24) also yields the shape of the phase boundary. From the condition $\xi^{-2}=0$ one finds $T_c\sim|\delta_0-\delta_0^{\mathrm{cr}}|^{\psi}$, where $\psi=\psi^{\mathrm{qc}}=\frac{z}{d+z-2}$ for quantum criticality, reproducing the result by Millis.[16] This value crosses over to $\psi=\psi^{\mathrm{tri}}=\frac{1}{2}$ when the quartic coupling approaches the tricritical value u_0^{tri}. Only for the special case $D_0'=0$ one obtains $\psi^{\mathrm{tri}}=\frac{z}{2(d+z)-4}$. For small $D_0'\neq 0$ a crossover between different exponents occurs.

The values of ψ^{qc} and ψ^{tri} can also be obtained using arguments based on a scaling relation of the free energy[4,14] and the exponents present in Eq. (24) too. A solution to the RG flow equations is, however, necessary to establish the condition Eq. (22) and all the coefficients that determine the tricritical and crossover temperatures.

As already stated, for $u_0<u_0^{\mathrm{tri}}$ a classical tricritical point exists at $T=T^{\mathrm{tri}}>0$. In Eq. (24) this manifests itself by a negative value of D_1. From the condition $\xi^{-2}=0$ we find the shape of the phase boundary above $T=T^{\mathrm{tri}}$ to follow $|\delta_0-\delta_0^{\mathrm{tri}}|\approx a|T-T^{\mathrm{tri}}|+b|T-T^{\mathrm{tri}}|^2$. This reproduces a mean-field result for classical tricritical points.[23]

From the solution for $\delta(\Lambda)$ we also read off the shape of the crossover line separating the Fermi-liquid and the quantum critical regimes. The condition for the occurrence of the quantum-disordered (Fermi-liquid) regime[16] is that the correlation length becomes of the order of the inverse upper cutoff before classical scaling is reached, that is, $\delta(\Lambda^*)>\Lambda_0^2$. From this condition we find the shape of the crossover line as $\delta_0-\delta_0^{\mathrm{cr}}\sim T^{2/z}$, as can also be deduced by a phenomenological reasoning.[1,2]

The linearized flow equations discussed above are not applicable in two dimensions, where the a_2 coefficient exhibits a logarithmic infrared singularity for $\xi\to\infty$ at $T>0$. The divergence is cured when one accounts for the renormalization of the quartic interaction coupling via the terms of order a_4^2.

IV. SUMMARY

We have presented a study of crossover behavior occurring in the vicinity of metallic quantum critical and tricritical points. The analysis is based on renormalization-group flow equations applied to the Hertz action, retaining a ϕ^6 term. It complements and extends an earlier work (Ref. 14), providing results in the quantum critical regime and delivering analytical insights. We focused on crossovers occurring in the temperature dependence of the correlation length and the shape of the phase boundary in the quantum critical regime above the quantum critical and tricritical points in the phase diagram. The linearized form of the flow equations could be solved analytically in $d>2$, yielding exponents for the power laws obeyed by the correlation length and phase boundary. In the quantum tricritical regime the power-law contribution to the inverse correlation length generated by the ϕ^6 interaction provides only a subleading correction to the generic quadratic temperature dependence induced by the temperature dependence of the coefficients in the bare action. For quantum critical systems close to quantum tricriticality, we computed the crossover temperature above which tricritical scaling is observed. In the situation where a tricritical point occurs at $T>0$, we obtained an expression for the tricritical temperature and recovered the shift exponent corresponding to the classical theory of tricriticality.

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APPENDIX: EXPLICIT EXPRESSIONS FOR THE CONSTANTS IN SEC. III

In this appendix we give explicit expressions for the constants appearing in the solutions to the flow equations in Sec. III. For the equations for $u(\Lambda)$ we have introduced

$$C = A\Lambda_0^z, \quad C' = \Lambda_0^{2-d}\left(\frac{2\pi Z_\omega}{Z}\right)^{(d-2)/z}\left(B - \frac{2\pi Z_\omega}{Z}A\right). \tag{A1}$$

For $\delta(\Lambda)$ one has

$$D_0 = \delta_0\Lambda_0^2 + A'\left(u_0 + \frac{1}{2}v_0 A\Lambda_0^z\right)\Lambda_0^{z+2}, \tag{A2}$$

$$D_1 = \Lambda_0^{4-d}(u_0 + v_0 A\Lambda_0^z)\left(\frac{2\pi Z_\omega}{Z}\right)^{(d-2)/z}\left[B' - A'\left(\frac{2\pi Z_\omega}{Z}\right)\right], \tag{A3}$$

and

$$D_2 = \frac{A' A\Lambda_0^{6-2d}}{2}\left(\frac{2\pi Z_\omega}{Z}\right)^{[2(d+z)-4]/z} + B'\Lambda_0^{6-2d}\left[\frac{B}{2}\left(\frac{2\pi Z_\omega}{Z}\right)^{(2d-4)/z} - A\left(\frac{2\pi Z_\omega}{Z}\right)^{(2d+z-4)/z}\right], \tag{A4}$$

where

$$A = 120 v_d \frac{Z}{\pi Z_\omega}\frac{1}{(d+z-2)^2}, \tag{A5}$$

$A' = 96A/120$ and

$$B = \frac{120 v_d}{d(d-2)}, \quad B' = \frac{96 v_d}{d(d-2)}. \tag{A6}$$

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